A data-driven method for robust fault diagnosis

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Abstract
There has been a growing concern over the security and reliability of the dynamic system among the people. An effective fault diagnosis and fault-tolerant control technology is of prime importance for the strong support on the long lifetime and reliable operation of the modern control system. A data-driven design method for robust fault diagnosis is proposed for discrete linear time-invariant. Based on the equivalent space of off-line data identification and H2 performance index, residual generations for robust fault detection, robust reduced order fault detection, robust fault isolation and robust fault identification are constructed for sensor and actuator faults, respectively. Compared with the existing data-driven design method of fault diagnosis, the designed residual signal in this paper has high sensitivity to faults and disturbances to robustness and thus improves the effectiveness of the fault diagnosis system.
Key words: DYNAMIC SYSTEM, DATA, ROBUSTNESS, FAULT

Introduction

Fault diagnosis technology based on the analytical model has aroused great concerns in academic and industrial circles since the 1970s, especially a variety of methods that have been derived from different theoretical platforms for linear time-invariant system [1-3]. These methods need to know the analytical model of the monitored object. Consequently, establishing an accurate mathematical model through the mechanism analysis is a requirement of fault diagnosis method based on the analytical model. However, with the rapid development of information science and technology, it takes engineers a large amount of time and effort to apply the traditional mechanism modeling method into the large-scaled and complicated multivariable systems. Unlike the mechanism modeling, system identification, especially subspace identification technique [4] provides the application basis for fault diagnosis technology based analytical model by identifying the system state matrix.

With the constant development and deepening of data-driven theory, some scholars have proposed the data-driven fault diagnosis method recently. Different from traditional system identification, the identification of system state matrix is not required. Thereby, the design cost of fault diagnosis system can be greatly simplified and the reliability can be improved. Based on replacing system matrix by Hankel matrix in the subspace identification process, literature [5] directly builds the system output estimator to achieve fault diagnosis. Other than the above method, literature [6] proposes a data-driven design method for robust fault diagnosis system, namely, through directly designing the pricing space and vector based on the subspace identification, the residual generator for fault diagnosis can be designed. The literature [7] makes further research and put forward the pricing space and vector based on the subspace identification and the method of residual generation. Though above literatures have studied the design of data-driven fault diagnosis system, the designed residual robustness hasn't been taken into consideration. If the sensor fault and actuator disturbance exist in the system simultaneously (or, the other way around), the misreporting and underreporting phenomena would occur in the designed fault diagnosis system of the above literatures.

In allusion to the fault diagnosis system design of discrete linear time-invariant system, this paper analyzes the floating-point operations based on the equivalent space method of subspace identification and compares with other subspace identification methods. A data-driven design method for robust fault diagnosis system has been put forward on the basis of the identified equivalent space. Based on the equivalent space of off-line data identification, this method designs the H2 performance index and residual generations for robust fault detection, robust reduced order fault detection, robust fault isolation and robust fault identification are constructed for sensor and actuator faults, respectively.

Data-driven Equivalent Space Design

In this section, the general discrete linear time-invariant system is taken into consideration. The traditional function model is:

$$y(z) = G_u(z)u(z) + G_w(z)w(z) + v(z)$$

The minimal realization of state function model is:

$$x(k+1) = Ax(k) + Bu(k) + w(k)$$

$$y(k) = Cx(k) + Du(k) + v(k)$$

Of which, $u \in R^I$, $y \in R^m$, $x \in R^n$ represents control input, measurement output and system state, respectively. $w \in R^n$, $v \in R^m$ represent system process noise and measurement noise, respectively. Assume that w and v are two irrelevant white noises with mean value of zero, the variance would be. R_w, R_v Assume that the system is observable and controllable, A, B, C, D jointly represents a state matrix and the traditional function models are as below, respectively:

$$G_u(z) = C(zI - A)^{-1} B + D \tag{1}$$

$$G_w(z) = C(zI - A)^{-1} \tag{2}$$

Assume that there is a system matrix A, B, C, D in the system, both the system order n and noise information R_w, R_v are unknown, while the system input and output $\{u, y\}$ are valid.

In consideration of the input and output data of the open-loop collection system, the two groups of typical output Hankel matrix are:

$$Y_f = \begin{bmatrix} y(k) & y(k+1) & \cdots & y(k+N-1) \\ y(k+1) & y(k+2) & \cdots & y(k+N) \\ \vdots & \ddots & \ddots & \vdots \\ y(k+s) & y(k++s+1) & \cdots & y(k+s+N-1) \end{bmatrix} \in R^{msfXN} \tag{3}$$

$$Y_p = \begin{bmatrix} y(k-s-1) & y(k-s) & \cdots & y(k+s+N-2) \\ y(k-s) & y(k-s+1) & \cdots & y(k+s+N-1) \\ \vdots & \ddots & \ddots & \vdots \\ y(k-1) & y(k) & \cdots & y(k+N-2) \end{bmatrix} \in R^{msfXN} \tag{4}$$

Of which, Y_f, Y_p represent "future" output Hankel matrix and "past" output Hankel matrix, respectively. Parameter N represents the sample quantity, s represents the order of equivalent vector and sample of satisfies s_f, $s_f = s+1 > n$ (now that the system order n is unknown, the order sf needs to be set big enough).

The specific steps of the algorithm are as follows:

Step 1: collect data Z_f and Z_p to construct the $\frac{1}{N} Z_f Z_P^T$.

Step 2: conduct SVD decomposition for $\frac{1}{N} Z_f Z_P^T \cdot \frac{1}{N} Z_f Z_P^T = U_z \begin{bmatrix} \Sigma_{z,1} & \\ & 0 \end{bmatrix} V_z^T$,

$U_z = \begin{bmatrix} U_{z,11} & U_{z,12} \\ U_{z,21} & U_{z,22} \end{bmatrix}$ Of which, $U_{z,12} \in R^{msfX(msf-n)}$, $U_{z,22} \in R^{IsfX(msf-n)}$.

Step 3: construct equivalent space $\Gamma_s^{\perp} = U_{z,12}^T$ and relevant matrix $\Gamma_s^{\perp} H_{s,M} = -U_{z,22}^T$.

Note 2.1: the step 3 of the algorithm 2 is constructed by conducting SVD decomposition.

$$\frac{1}{N}\left[U_{z,12}^T U_{z,22}^T\right] Z_f U_P^T = \frac{1}{N}\begin{bmatrix} U_{z,11} & U_{z,12} \\ U_{z,21} & U_{z,22} \end{bmatrix}\begin{bmatrix} \Sigma_{z,11} & \\ & 0 \end{bmatrix} V_z^T = 0 \quad (5)$$

$$\Rightarrow \frac{1}{N}\left[U_{z,12}^T U_{z,22}^T\right] Z_f U_P^T = \frac{1}{N}\left[U_{z,12}^T U_{z,22}^T\right]\begin{bmatrix} \Gamma_s & H_{u,s} \\ 0 & I \end{bmatrix}\begin{bmatrix} X(i) \\ U_f \end{bmatrix} U_P^T = 0 \quad (6)$$

In order to ensure the validity of step 3, the input serial numbers U_f, U_p need to satisfy the sufficient incentive condition [111].

$$rank\left(\frac{1}{N}\begin{bmatrix} X(k) \\ U_f \end{bmatrix} Z_p^T\right) = N + ls_f \quad (7)$$

Data-driven Design for Residual Generator of Robust Fault Diagnosis

This section studies the fault diagnosis of the following system models:

$$x(k+1) = Ax(k) + B\left(u(k) + f_a(k)\right) \quad (8)$$

$$y(k) = Cx(k) + D\left(u(k) + f_a(k)\right) + f_b(k) \quad (9)$$

Of which, if $f_a \in R^I$ represents the additive fault of the actuator, then $f_b \in R^m$ represents the additive disturbance of the actuator and $f_b \in R^m$ represents the additive fault of the sensor.

The equivalent space model is represented as:

$$y_s(k) = \Gamma_s x(k-s) + H_{s,u} u_s(k) + H_{s,w} f_{s,a}(k) + f_{s,b}(k) \quad (10)$$

Of which, $y_s(k) = \left[y^T(k-s)\, y^T(k-s+1) \cdots y^T(k)\right] \in R^{msf}$. $u_s(k) \in R^{Isf}$, $f_{s,a}(k) \in R^{Isf}$ and $f_{s,b}(k) \in R^{msf}$ can be defined in a similar way.

Assume that the equivalent space $\Gamma_s^{\perp}$ is obtained through the algorithm, select the equivalent vector to conduct left multiplication to the both sides of the equation and the residual generator based on the equivalent space can be obtained.

$$r(k) \triangleq v_s y_s(k) - v_s H_{s,u} u_s(k) = v_s f_{s,b}(k) + v_s H_{s,u} f_{s,a}(k) \quad (11)$$

It can be seen from the above equation, the key of designing the residual generator based on the equivalent space and achieving robust fault diagnosis lies in how to select vector Vs from the equivalent space $\Gamma_s^{\perp}$, decreasing disturbance and strengthening the impact on residual by the fault. According to the different uses of fault diagnosis, the residual generations for robust fault detection, robust reduced order fault detection, robust fault isolation and robust fault identification are constructed for sensor and actuator faults, respectively the following four sections.

In conclusion, the residual generations of the following fault detection for sensor and actuator faults are presented as below:

$$r_b(k) = v_{s,b} y_s(k) - \beta_{s,b} u_s(k) \quad (12)$$

$$r_a(k) = v_{s,a} y_s(k) - \beta_{s,a} u_s(k) \quad (13)$$

If $s \to \infty$, $\lambda_{s,\min}$ and $\lambda_{s,\max}$ would be converged to the H2 index. As a consequence, if the minimum eigenvalue $\lambda_{s,\min}$ or minimum eigenvalue $\lambda_{s,\max}$ of the given order isn't satisfied with the requirement, the order of given space can be appropriately increased to improve the generated robust performance by the residual and increase the online calculated quantity. Selecting the order of equivalent space should weigh both the robustness of residual and the online calculated quantity. Through above analysis, the algorithm is given of robust fault residual generator for sensor fault as below.

Step 1: collect data Z_f, Z_p and construct the equivalent space $\Gamma_s^{\perp}$ and $\Gamma_s^{\perp} H_{s,u}$ based on the algorithm G. 1.

Step 2: obtain the minimum eigenvalue $\lambda_{s,\min}$ and eigenvector $l_{s,\min}$.

Step 3: obtain the robust equivalent vector $v_{s,b}$ and $\beta_{s,b}$.

Step 4: construct the residual generator of robust fault detection based on the equivalent space.

The algorithm of residual generator design of robust fault detection for actuator fault is given as below:

Step 1: collect data Z_f, Z_p, and construct the equivalent space $\Gamma_s^{\perp}$ and $\Gamma_s^{\perp} H_{s,u}$, based on the algorithm G. 1.

Step 2: obtain the minimum eigenvalue $\lambda_{s,\min}$ and eigenvector $l_{s,\min}$.

Step 3: obtain the robust equivalent vector $v_{s,b}$ and $\beta_{s,b}$.

Step 4: construct the residual generator of robust fault detection based on the equivalent space.

The residual generator design methods of data-driven fault detection and robust reduced order fault detection are introduced respectively in the former. However, these methods only could reflect whether there are faults in the sensor and actuator for MIMO system and the fault location cannot be identified. Therefore, the data-driven robust discrete residual generator for sensor and actuator has been studied. Considering the residual generator based on equivalent space, if there has disturbance, the fault isolation problems for No. J sensor and actuator are described as below, respectively:

$$r_{b,iso,j} = v_{b,iso,j} f_{b,j} \quad (14)$$

$$r_{a,iso,j} = v_{a,iso,j} f_{a,j} \quad (15)$$

The main mission for robust fault isolation in this section is to select a group of equivalent vectors from the identified equivalent space $\Gamma_s^{\perp}$ and $\Gamma_s^{\perp} H_{s,u}$ and build the residual generator groups for robust fault isolation. The following content takes I residual generator for the robust fault isolation as an example for generalizing the whole group of residual generator for the robust fault isolation.

First of all, to extend the equivalent space as:

$$\Gamma_s^{\perp} = \left[\Gamma_{s,0}^{\perp} \cdots \Gamma_{s,s}^{\perp} \right] \quad (16)$$

$$\Gamma_{s,i}^{\perp} = \left[\Gamma_{s,i,1}^{\perp} \cdots \Gamma_{s,i,m}^{\perp} \right] \in R^{\eta X m} \quad (17)$$

$$\Gamma_{s,i,j}^{\perp} = R^{\eta}, i = 0, \cdots s, \; j = 1, \cdots, m \quad (18)$$

To extend $\Gamma_s^{\perp}$ and $\Gamma_s^{\perp} H_{s,u}$ in a similar way:

$$E_s = \left[E_{s,0} \cdots E_{s,s} \right], E_{s,i} = \left[E_{s,i,1} \cdots E_{s,j,I} \right] \in R^{\eta XI} \quad (19)$$

$$E_{s,i,j} \in R^{\eta}, i = 0, \cdots, s, \; j = 1, \cdots, l \quad (20)$$

Experimental Analysis

This section conducts simulation verification for the effectiveness of proposed data-driven system method for robust fault diagnosis. Taking CSTH (continuous stirred tank heater) chemical system as an example as shown in Figure 2-1. The main water channel is used for mixing hot water and cold water, heat flow in the coil pipe is used for heating the mixed water in the container and the long pipe below is used for draining away water. As the MIMO non-linear control system coupled with the liquid level and the temperature, the two latter things are required to maintain within a reasonable range. There are inlet valve for cold water ui, valve for coil pipe u and valve for hot water u3 in the actuator, measurement pipe displacement Yi, liquid level Y and temperature Y3 in the sensor. The system state mainly includes the volume of mixed water xi, direct coefficient in the system and water inflow x3. The above input and output signals are unified as electrical signals within a range of 4-20mA.

Assume that the model parameters of CSTH system are unknown, the system input and output data are preprocessed, uniformly distribute in the sample space and satisfy the integrity. With the given sampling time of 1 second, select the forged random binary input vector for open loop experiments and collect N=1000 groups of input and output data. Select equivalent order S=6 and construct the Hankel matrix based on the algorithm identification equivalent space $\Gamma_s^{\perp} \in R^{18\times21}$ and $\Gamma_s^{\perp} H_{s,u} \in R^{18\times21}$.

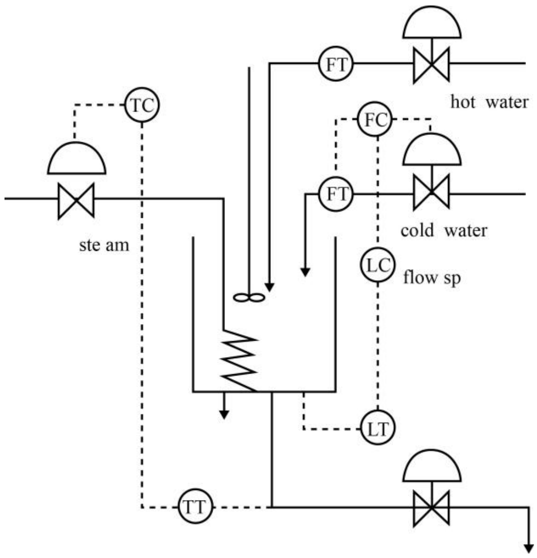

Figure 1. CSTH Schematic Diagram

Table 1 shows the time of the algorithm N4SID, MOESP, CVA and the extended observable matrix Γ_s of algorithm calculation CSTH system under the configuration (Intel Core i3-2130 Processor, Windows 7 64-Bit Home Edition and Matlab R2009a). The results shown in Table 1 verified the superiority of in the calculated amount.

algorithm	Main order	Time (second)
N4SID	$(6sl+4sm+2)l$	0.2622

MOESP	$(10sl+6sm+3)$	0.3492
CVA	$(12sl+8sm+4)$	0.3607
algorithm 2.1	$2s^2(l+m)^2 N$	0.0083

Figure 2 and 3 represent the time variation curves of the robust fault detection residual for sensor fault and the worst robust fault detection residual for sensor fault, respectively. It can be seen from the simulation diagram, taking the robust fault detection of actuator fault and assume that there has periodical disturbance in the sensor y1 with value of $f_{b,1}(k)=0.1\cos\left(\frac{\pi}{25}k\right)$, then there has square wave fault $f_{a,3}(k)$ with amplitude of 0.5, period of 314 and proportion of 30% in step 300 in the actuator u3. Figure 4 and 5 represent the time variation curves of the robust fault detection residual for actuator fault and the worst robust fault detection residual for actuator fault, respectively. It can be seen from the simulation diagram, the robust fault detection residual can effectively detect the actuator fault when there exists disturbance in the sensor.

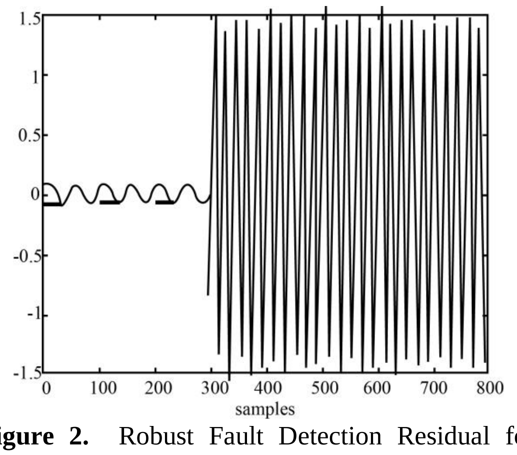

Figure 2. Robust Fault Detection Residual for Sensor Fault

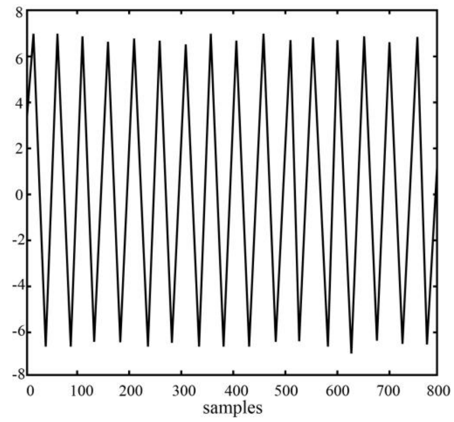

Figure 3. Worst Robust Fault Detection Residual for Sensor Fault

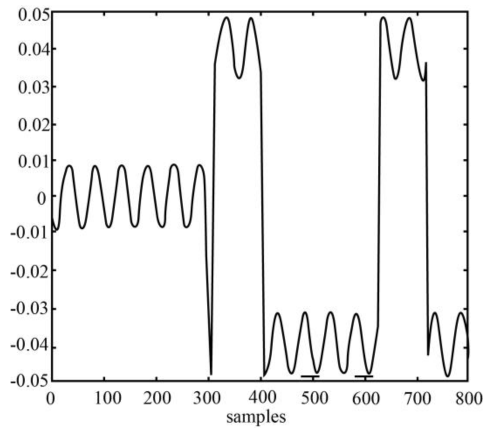

Figure 4. Robust Fault Detection Residual for Actuator Fault

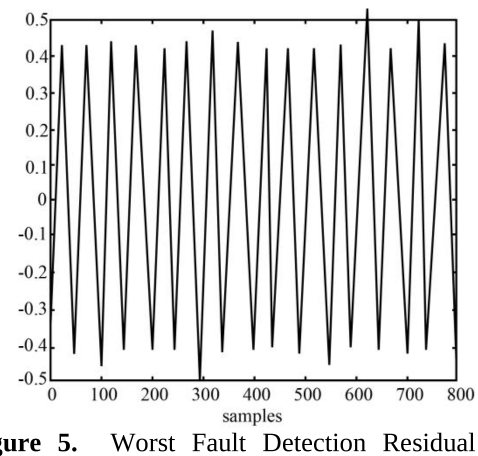

Figure 5. Worst Fault Detection Residual for Actuator Fault

The same combination of fault and disturbance of robust fault detection is given and Figure 6 and 7 are time variation curves of robust reduced order fault detection residual for sensor fault and worst reduced order fault detection residual for sensor fault. Figure 8 and 9 are time variation curves of robust reduced order fault detection residual for actuator fault and worst reduced order fault detection residual for actuator fault. It can be seen from the simulation diagram, the robust reduced order fault detection residual could not only maintain the performance of the robust fault detection, but also could reduce the online calculated amount generated by the reduced residual.

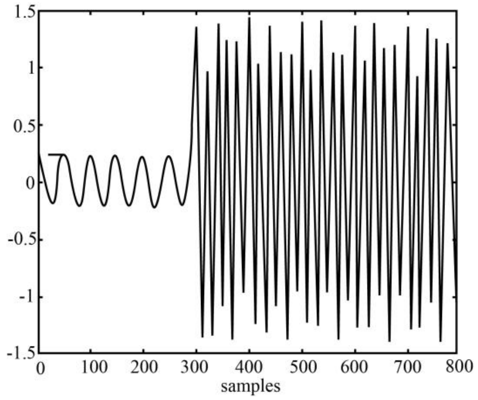

Figure 6. Robust Reduced Order Fault Detection Residual for Sensor Fault

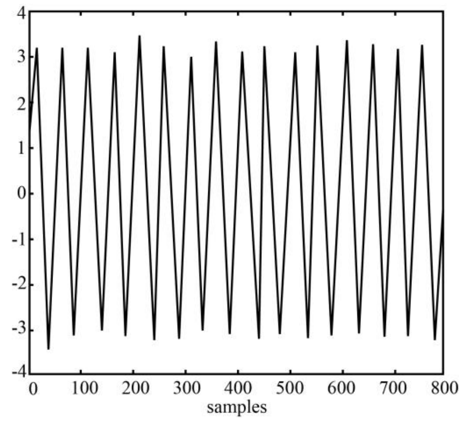

Figure 7. Worst Reduced Order Fault Detection Residual for Sensor Fault

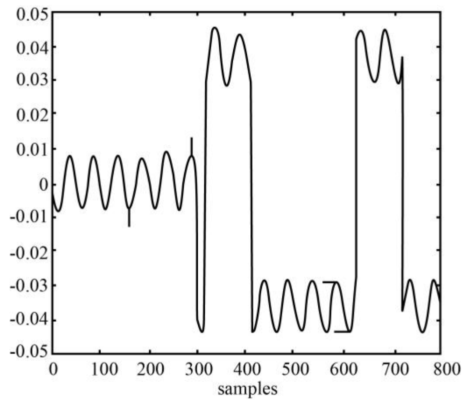

Figure 8. Robust Reduced Order Fault Detection Residual for Actuator Fault

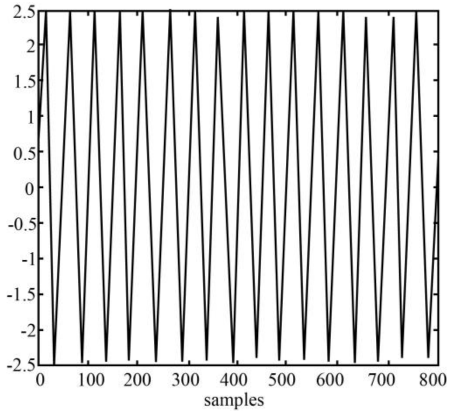

Figure 9. Worst Reduced Order Fault Detection Residual for Actuator Fault

Taking the robust fault isolation of actuator fault and assume that there has periodical in the sensor y1 with value of, $f_{a,1}(k)=\sin\left(\frac{\pi}{25}k\right)+\cos\left(\frac{\pi}{10}k\right)+\sin\left(\frac{\pi}{5}k\right)$ then there has sudden fault in step 300 in the actuator u3 with value of $f_{b,1}(k)=-0.02k+6$. Figure 10 is the time variation curve of comparing the robust fault isolation and the worst fault isolation residual in the sensor fault. It can be seen from the simulation diagram, compared with the worst fault isolation residual, the robust fault isolation has better sensitivity when there has disturbance in the actuator.

Taking the robust fault isolation of actuator fault and assume that there has periodical disturbance in the sensor y1 with value of $f_{b,1}(k)=0.1\sin\left(\frac{\pi}{25}k\right)$, then there has sudden fault in step 300 in the actuator u3 with value of. $f_{a,1}(k)=10$ Figure 11 is the time variation curve of comparing the robust fault isolation and the worst fault isolation residual in the sensor fault. It can be seen from the simulation diagram, compared with the worst fault isolation residual, the robust fault isolation has better sensitivity when there has disturbance in the actuator.

Because there's no direct input item in the SCTH system, i.e. D=0, according to Note 2.4, the algorithm proposed in the following verification cannot use equivalent space for actuator fault identification.

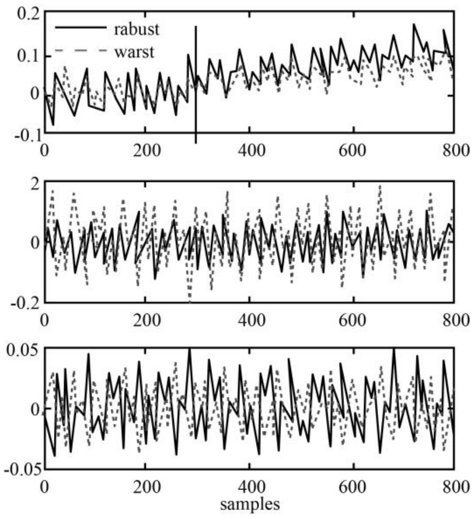

Figure 10. Fault Isolation Residual for Sensor Fault

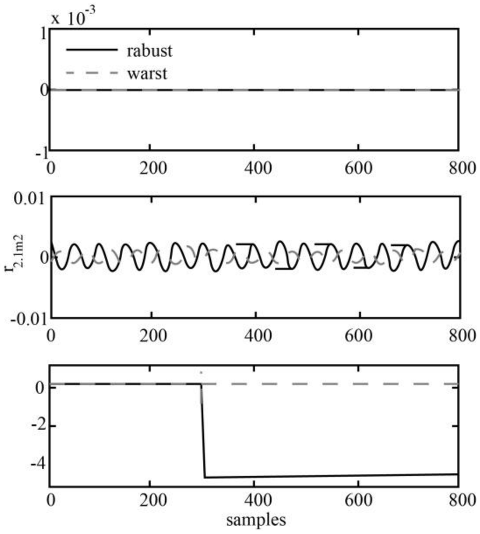

Figure 11. Fault Isolation Residual for Actuator Fault

Assume that there exists periodical disturbance in the actuator u3 with value of $f_{a,3}(k)=0.1\left(\cos\left(\frac{\pi}{25}k\right)+\sin\left(\frac{\pi}{10}k\right)+\sin\left(\frac{\pi}{5}k\right)\right)$ and the sudden fault in the sensor Y2, the time variation curve are shown in Figure 12.

Figure 12 is the time variation curve of robust isolation residual for sensor fault and Figure 11 is the time variation curve of fault isolation residual for actuator fault. It can be seen from the simulation diagram, the robust fault identification can achieve the robustness fault identification for sensor fault when there exists disturbance in the actuator.

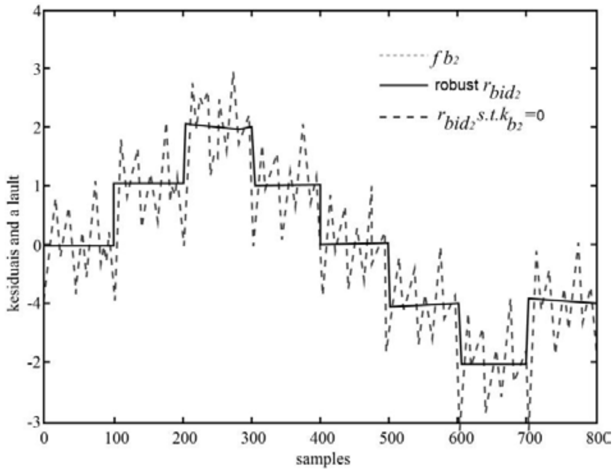

Figure 12. Fault Identification Residual for Sensor Fault

Conclusions

This paper proposes a data-driven design method for the robust fault diagnosis system through conducting a data-driven fault diagnosis research. According to the equivalent space of off-line data identification and H2 performance index, this method designs the residual generation based on the equivalent vector and robust fault detection for sensor and actuator faults. Considering the online calculated amount of residual generator based on the equivalent vector, the robust reduced order fault detection has been designed. Considering the fault isolation of the sensor and actuator, robust fault isolation has been designed. Considering the fault identification for faults of sensor and actuator, the robust fault identification has been designed. The numerical simulation shows that compared with the existing data-driven system design method for fault diagnosis, the designed residual signal in this paper could better reflect the fault data and reduce the disturbance of perturbation as well as improve the effectiveness of the fault diagnosis.

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